

for original papers of real importance; but the task of recording progress, of acting as the historians of science, is rightly declined by societies which aim at advance rather than at retrospect. Hence this most important function can be best discharged by these great Associations, and it will always suffice to save them from degenerating into scientific camp-meetings or picnics.

The Sections in the American Association are equally numerous with those in the British at the present time, though differently arranged. Mathematics and Physics are divided, Geology and Geography united; Histology and Microscopy form a section separate from Biology. We doubt the advantages of the union in the second case, and of the separation in the third. That no address is printed in this volume, and that the only record of the proceedings of the Section of Histology and Microscopy is the statement that, although some meetings took place, no papers were read before it, seems an indication that, as in Britain, its subjects might safely be merged in Biology, the latter Section having the power of temporary subdivision.

In another respect too the American differs from the British Association. In the latter the delivery of an address is the first official act of its President, in the former it is the last. The address at Minneapolis was delivered by Principal (now Sir William) Dawson, and is characterised by the scientific caution and literary ability of its author. It gives a critical sketch of the results of geology, more especially with reference to the development of the earlier rocks and to the evolution of living creatures. In regard to the former, Sir W. Dawson inclines to drawing a marked line of separation between the Lower Laurentian or Ottawa gneiss of Sir W. Logan and the Middle Laurentian or the Grenville series of the same, which is characterised by beds of limestone and dolomite, "quartzite, quartzose gneisses, and even pebble beds," besides graphite, iron ore, and the debatable *cozoon*, which Sir W. Dawson considers as indicating the existence of land surfaces of the fundamental gneiss. The Upper Laurentian or Norian series is noticed with due caution, though it is regarded as decidedly younger than the preceding formation. The Huronian, Montalban, and Taconian (Lower Taconic of Emmons) are next mentioned, but the author, though inclining to the views of Dr. Sterry Hunt as to their order of succession, forbears to dogmatise as to their precise relations either mutually or with "certain doubtful deposits around Lake Superior." With regard to the development of life, he is decidedly adverse to the evolution school among biologists, but is not able to add anything material to the familiar arguments of its opponents. The address concludes with a brief notice of some of the obscure markings, variously referred by palæontologists to algæ, protozoa, and tracks of various animals, and with a critical sketch of the theories relating to the Glacial Epoch, in which he expresses himself as opposed to the extreme views of the former extension of land-ice and its erosive action which are favoured by some geologists.

Two other papers are given as "read in General Sessions," which we presume may be regarded as in some respect analogous with the evening discourses at the British meetings. The one by Dr. Sterry Hunt, "On a Classification of the Natural Sciences," is printed in abstract

only; the other, by Prof. E. D. Cope, entitled "The Evidence for Evolution in the History of the Extinct Mammalia," is an extremely able and temperate sketch of the views antagonistic to those entertained by the retiring President. "The German Survey of the Northern Heavens" forms the subject of an interesting address by Prof. W. A. Rogers, who presided over the section of Mathematics and Astronomy, and Prof. H. A. Rowland delivered a "Plea for Pure Science" to the section of Physics. Both these sections received a considerable number of communications. The section of Chemistry does not appear to have had a special address, and the number of papers read before it was not large. The same may be said of the Mechanical Section, in which only seven papers are recorded as read. Prof. Hitchcock, in the section of Geology and Geography, took the "Early History of the North American Continent" as the subject of his address, in which he favours the idea that the bulk of the early crystalline rocks are of igneous origin, being metamorphosed volcanic rocks or tuffs. Ice and its leavings form the subject of a large proportion of the papers read before this section. More than one of these is of much interest, especially that by Mr. W. Upham on the Minnesota Valley in the Ice Age. Messrs. H. C. Bolton and A. A. Julien describe "The Singing Beach of Manchester, Mass.," noticing in the course of the paper the sonorous sand in the Island of Eigg (Hebrides), as well as others on record. It results from their observations that the sound is due to the grains, which are not rounded, but have flat and angular surfaces. It is, we think, undoubtedly a vibration phenomenon. We are acquainted (probably the fact is common) with a small screw-tap in a lavatory, which is loudly sonorous when a certain amount of water is allowed to issue, but silent in other positions. Prof. W. J. Beal, in addressing the Section of Biology, deals with "Agriculture, its Needs and Opportunities;" and the Section received a considerable number of interesting communications. Dr. Franklin B. Hough addressed the Economic Section on the method of statistics, and the address of Mr. E. B. Elliott, delivered to the same Section at the preceding Montreal meeting, is printed in this volume. This Section does not appear to receive nearly so many communications as the corresponding one of the British Association. The address of Prof. O. T. Mason to the Section of Anthropology deals with the scope and value of anthropological studies, and a considerable number of interesting papers were read. Those relating to mound-building may be of service to European archaeologists as offering suggestions which may help in the interpretation of some of the earthworks in the Old World.

#### LETTERS TO THE EDITOR

*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts. No notice is taken of anonymous communications.*

*[The Editor urgently requests correspondents to keep their letters as short as possible. The pressure on his space is so great that it is impossible otherwise to insure the appearance even of communications containing interesting and novel facts.]*

#### On the Terminology of the Mathematical Theory of Elasticity

THE late Dr. Todhunter left, in an incomplete state, a valuable "History of the Mathematical Theories of Elasticity,"

which the syndics of the Cambridge University Press have intrusted to me to complete and edit. In reading the great number of memoirs relating to the subject I have been much struck by the want of a clear and accurate terminology in both theoretical and practical elasticity. I have been forced to the conclusion that the great discrepancy, which is often to be found between theoretical and practical results, is in some measure due to the want of this terminology (e.g. the extreme looseness of the term "limit of elasticity"). I find it needful for the purposes of the above work to adopt such a terminology, but before doing so it would be extremely valuable to have the opinion of some of our leading elasticians on the terms I venture to propose. I should be very glad of any suggestions, through the columns of NATURE, towards a definite and uniform terminology.

I am particularly dissatisfied with the term "limit of superimposition." It is exceedingly clumsy. Other possible terms are—"limit of superposable stress," "linear limit," and "limit of constant slope," the last two phrases having reference to the fact that the stress-strain curve at this limit ceases to be a straight line. With regard to this limit of superimposition I may remark that it may arise from one of two causes—(1) The strain components become so large that we cannot neglect the squares of small quantities, or the stress components can no longer be taken proportional to those of strain. This might happen before permanent set. (2) Permanent set may arise which does not follow the generalised Hooke's Law. This seems the more probable case, and has been adopted below. Prof. Kennedy tells me that he thinks when a body has been reduced to a state of ease that the superior elastic limit and the limit of superimposition coincide.

It has been proposed, I believe, to term that limit of stress at which bars begin locally to "thin down" the limit of viscosity. The "limit of uniform strain" is not altogether satisfactory or quite suggestive of this peculiar viscosity. "State of maximum stress" might perhaps serve the purpose, were one quite sure that this state always coincides with the viscous limit.

In the following remarks I have been much assisted by Prof. J. Thomson's epoch-making paper in the *Cambridge and Dublin Mathematical Journal* for 1848, and even more by Prof. Alex. B. W. Kennedy's paper on Riveted Joints in the *Proceedings of the Institution of Mechanical Engineers*, April, 1881 (especially pp. 208-213).

We have first to distinguish between two classes of materials. In the one we may suppose the particles to be in a state of internal stress before any external force is applied; in this case any, the least, external stress will probably produce permanent set. If this stress be removed and then reapplied, after one or two trials it will cease to produce permanent set, or at least the permanent set will be extremely small as compared with the elastic strain. We thus need a term to mark that state of the body when external stress does not produce permanent set owing to the existence of internal stress. This might perhaps be termed the *state of ease*. Many discordant results with regard to the constants of elasticity are not improbably due to the fact that the ratio of stress to strain has been measured before the material has been reduced to a *state of ease*. In the second class of materials we may suppose this state of ease to exist before the application of any stress. Supposing a body to be in its state of ease, there will then exist two limits, one on one side, and one on the other of the unstrained shape, which may be termed the *inferior and the superior limits of perfect elasticity*. Any external stress which does not produce a strain exceeding these limits will not give rise to permanent set. These inferior and superior limits of perfect elasticity mark, as a rule, the range covered by the usual mathematical theory. Within these limits it is generally safe to assume that the components of internal stress are proportional to the components of strain. In some cases, i.e. cast iron, where, however, it is difficult to produce the state of ease, this does not seem to be accurate—the stress and strain components appear never to be proportional.

In most materials the range of perfect elasticity is not large. An external stress, which is by no means nearly equal to that which is required to produce rupture will give rise to a permanent set. Thus permanent set in some materials will commence at a stress only  $\frac{1}{3}$  to  $\frac{1}{4}$  of the stress that those materials are capable of standing. Thus beyond the limit of elasticity we have first a range of stresses, which produce strains partly elastic and partly permanent. The strain in this range might still remain proportional to the stress; the permanent is yet small as com-

pared with the elastic part of the strain. This range is bounded, however, by a stress for which the strain ceases to be proportional to the stress. In other words, the "generalised" Hooke's law is no longer applicable. Up to this point, if we are merely desirous of finding the strain produced by any system of statical stress, the mathematical equations of elasticity will apply, supposing, as seems probable, that the elastic constants do not alter, owing to the permanent set. Those equations would not of course be valid if we wished to find the strain in the body, if the stress were altered, nor would they suffice to treat vibratory motions capable of producing permanent set. This limit, which is that at which the *ut tensio sic vis* principle ceases, requires a name. It might perhaps be termed the *limit of superimposition*. That is to say, if a certain addition to this limiting stress produced a certain increase of strain, and a second addition another increase, these increments of stress, if superimposed would not produce the sum of the strain increments. It might at first sight appear more direct to term it the modular limit, or the limit of Hooke's law, but it would seem that, after this limit is passed, Hooke's law, probably with the same modulus, applies to so much of the strain as is elastic strain; in fact at the limit of superimposition it is the permanent set part of the strain which ceases to obey Hooke's law. In some materials the limits of perfect elasticity and of superimposition may coincide. At the latter limit the permanent set is still in some cases only one-twenty-fifth of the total strain. Neither of the limits above considered is *commercially* treated as the limit of elasticity. This is the point at which the material "breaks down," that is to say, the stress being continually increased, a strain is obtained which would be preserved by replacing the stress by one very much less. The material is unable to balance the stress upon it. If the stress be maintained the strain will suddenly increase by a considerable amount (without the stress being increased). This remarkable limit, it has been suggested by Mr. Tweddell, should be termed the *limit of fatigue*. The limit of fatigue being past, a small proportion of the strain, namely, so much as corresponds to the modulus, is elastic, the greater part is permanent set.

In the case of bars of iron subjected to longitudinal pull, if the stress be increased beyond the limit of fatigue, another limiting strain is reached, namely, one at which local contraction begins, or the bar commences to draw out at some point, i.e. the strain ceases to be uniform. The material now begins to act as if it were "viscous," and it would be convenient to describe this state as that of *viscosity*, had not this name been appropriated to that permanent set which may be produced by the application for a long period of a stress well within the limits of perfect elasticity. Closely associated, if not the same, with this *limit of uniform strain* is the state of maximum load. From this point onwards, as the strain increases the load decreases, till the breaking load is reached with a magnitude below that of the maximum load. To distinguish one from the other requires a special manipulation. As a rule, what is meant by the *absolute or breaking strength* is probably the *maximum load*, for if this load was allowed to remain, the bar would break under it. It might perhaps be convenient, however, to speak of one as the *maximum* and the other as the *terminal load*. With the terminal load the "elastic life" of the material is concluded. It must be remembered that owing to the bar locally thinning down, the stress per unit area at the terminal load is greater than the stress per unit area at the maximum load.

Such are the limits for which it is needful that a terminology should be established. I shall be extremely glad if any of the readers of NATURE, who happen to be elasticians, will suggest a more concise phraseology.

KARL PEARSON

University College, February 14

### Civilisation and Eyesight

I HAVE been interested in Lord Rayleigh's note on "Vision," and would offer my mite on the subject.

I have no doubt that brilliancy of image and power of distinguishing largely depend on definition. The brilliancy does so for the same reason as that which induces an artist to eighteen colour-effects by sharp contrasts. In the same way, if we seek to decide if two colours are alike, we place them in immediate contact with a sharp edge. Details are best seen with a telescope when the images are sharp and untroubled. When slight tremors are in the air, and the image is rapidly displaced in all directions, so that what we see is the resultant